SHOCK-WAVE INTERACTION WITH A CASCADE OF PLATES

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Singularities of the problem of shock interaction with permeable obstacles, from the practical to the purely scientific, attract the attention of researchers. In great part the available bibliography refers to shock interaction with wire meshes, i.e., obstacles that have no spatial extent in the direction of shock propagation.

The problem of nonstationary flow of a gas that occurs during incidence of a plane shock on a homogeneous set of semiinfinite prisms is examined. It is assumed that the characteristic features of the phenomenon occurring during shock incidence on a rigid permeable wall take place in this problem. The results of experiments yield a good basis for the development and perfection of theoretical models. In particular, comparison of the data obtained in experiment by the theory of Chester–Chesnell–Whitham and from a computation by the method of "coarse particles" results in a distinctive representation about the quality of the different approaches.

On the way to solving practical modifications of the problem of shock interaction with a permeable obstacle, we examine a simple and characteristic model which is a homogeneous set of prismatic plates (Fig. 1). The leading face of the set is parallel to the incident wave and a quasi-one-dimensional treatment is possible for the plane problem of nonstationary gas flow in such interaction. Let us note that the model is exhaustively described by one parameter

$$\alpha = 1 - \delta/(\delta + d), \tag{1}$$

where δ is the width of the gap between plates of thickness d.

Individual interaction between the shock and an element of the set occurs in the first stage of the process under consideration. Later, as the interference deepens, the circumstance that the composite is a system of many bodies turns out to be more and more essential. The uniformity of the element distribution specifies the closeness of the phenomenon in this phase to shock interaction with a straight wall of rigid material of permeability α (a homogeneous approximation). In another approach the problem is equivalent to the phenomenon caused by a shock in a channel passing the site (x_0) where its section is suddenly diminished so that the drop in the area corresponds to the magnitude of the parameter α. From the very beginning in the first case, and in the large time approximation in the second, the problem is self-similar (the length and time dimensionality parameters do not enter explicitly), and as experience in studying it shows [1-3], it allows an exact solution of the type "decay of a discontinuity" in the limit sense under the conditions that a momentum discontinuity is provided at the site where the channel section changes (x_0). The question of the content of the wave pattern originates. It is proved in [1] that under our conditions only the wave pattern displayed in Fig. 2 is possible (αe, reflected; αc, transmitted shock in the cascade; αd, contact surface; and αt, surface of discontinuity of the parameters at the site of the section change).

The magnitude of the discontinuity in the momentum is determined by secondary effects relative to the one-dimensional model. It transmits the dynamics of a two-dimensional flow in the neighborhood of x_0. Consequently, the problem becomes indeterminate within the framework of the one-dimensional model. Closure is possible by simplifying assumptions, e.g., an assumption about the isentropy of the flow in the neighborhood of x_0 [2] or (for small α) the assumption of weakness of the perturbations induced in the flow 3 by an obstacle [4]. A semiempirical approach is possible in which the indeterminacy is removed by taking account of information obtained in experiment.

The originality of the situation being expounded here is that it could be thought that experiment is necessary, then it is not necessary to switch the complex analytical apparatus for solving the system of equations of the decay of a discontinuity over to it. In fact, obtaining sufficient information for its automony in experiment (e.g., the flow parameters in zone 2) is not a simple matter, and it is consequently expedient to combine a simple incomplete experiment with complementary computations. On the other hand, it can be expected that

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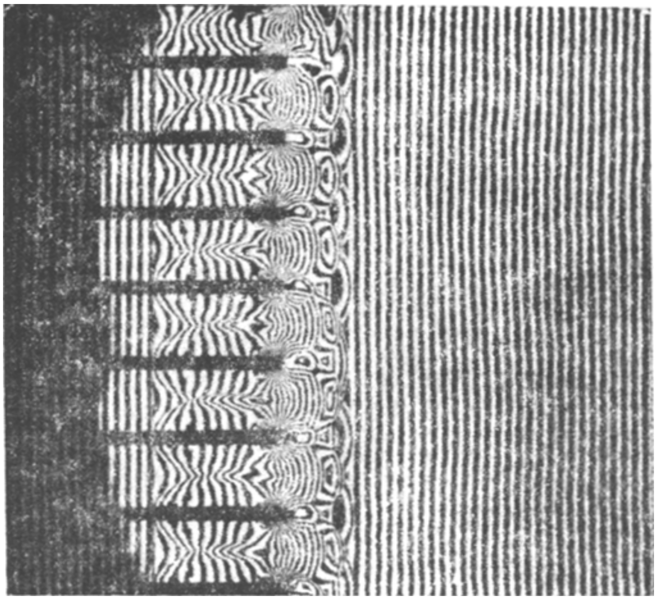

Fig. 1

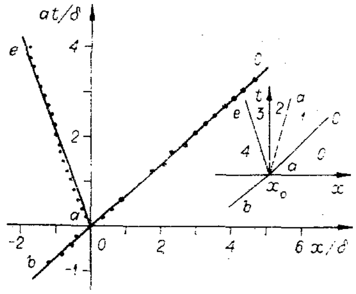

Fig. 2

the product of systematic experiments with the most characteristic modifications of the shape of the transition part can yield a foundation for the generalizations removing the necessity for experiment in each specific case. Evidently the model under consideration, the set of prismatic plates, can be acknowledged as sufficiently characteristic.

Investigation of the plane nonstationary gas flow originating during shock interaction with a cascade of plates was in an air shock tube for an incident shock Mach number in the range 1.2-2.0.

A specially developed separable model was used to assure the necessary variation in the parameter characterizing the permeability.

The quantity α has the value 0.09259, 0.1695, 0.2941, 0.3846, 0.5556, 0.7143. For a conception of the physical scales of the experiment, we mention that the thickness of the plates used is 2 mm. The shock tube is equipped with apparatus to measure the incident wave parameters, interferometry for the high-speed photorecording process, and synchronizing apparatus.

Instantaneous shadow photographs and interferograms of the flow field were obtained at definite times. An example of an interferogram obtained for the cascade characterized by the parameter $\alpha = 0.1695$ is presented in Fig. 1 ($\alpha = 0.1695$, M = 1.41, t = 23 μsec) for the interferometer adjusted to the "finite width" fringe.

In a number of cases, moving pictures are made by using a high-speed photorecorder to establish the approximateness of the approach with the central wave pattern. The wave pattern obtained in processing the high-speed motion pictures is presented in Fig. 2 ($\alpha = 0.1695$, M = 1.595). This result shows that the trajectories of the waves reflected by and entering the cascade differ slightly from straight lines. This not only indicates that the shocks entering and reflected by the cascade vary weakly in time but also that the influence of dissipative factors is slight in the scales considered, meaning that the problem is simulated well geometrically. To confirm this assertion, a series of additional experiments on shock interaction with cascades having the identical parameter α but different plate thicknesses was performed.

Analysis of the results obtained confirmed completely the possibility of geometric modeling. Differences in the velocities of the transmitted and reflected shocks for cascades with identical α but different plate thicknesses are within the limits of error of experiment and are random in nature.

Therefore, it was assumed in processing the experimental data that the phenomenon corresponds to central wave patterns of the type "decay of a discontinuity," and the result of measuring the location of the shock reflected from or penetrating the cascade at a known time can be used to determine the characteristic velocity of the corresponding wave. These data were later used in the system of relationships for the decay of the discontinuity:

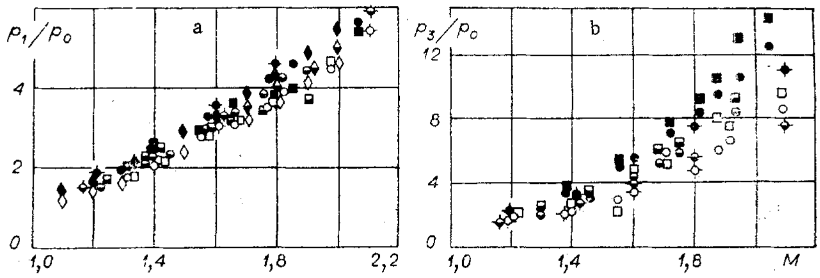

Fig. 3

$$
\begin{gathered}
q_4 = \frac{2}{\varkappa+1}\frac{U_0^2 - a_0^2}{U_0},\; p_4 = \frac{2\rho_0}{\varkappa+1}\left(U_0^2 + \frac{1-\varkappa}{2\varkappa}a_0^2\right),\\
\rho_4 = \frac{(\varkappa+1)\rho_0 U_0^2}{2a_0^2+(\varkappa-1)U_0^2},\; q_1 = \frac{2}{\varkappa+1}\frac{U_1^2-a_0^2}{U_1},\\
p_1 = \frac{2\rho_0}{\varkappa+1}\left(U_1^2 + \frac{1-\varkappa}{2\varkappa}a_0^2\right),\; \rho_1 = \frac{(\varkappa+1)\rho_0 U_1^2}{2a_0^2+(\varkappa-1)U_1^2},\\
q_3 = q_4 - \frac{2}{\varkappa+1}(V+q_4)\left[1-\frac{a_4^2}{(V+q_4)^2}\right],\\
p_3 = \frac{2\rho_4}{\varkappa+1}\left[(V+q_4)^2 + \frac{1-\varkappa}{2\varkappa}a_4^2\right],\; \rho_3 = \frac{(\varkappa+1)\rho_4(V+q_4)^2}{2a_4^2+(\varkappa-1)(V+q_4)^2},\\
(1-\alpha)\rho_2 q_2 = \rho_3 q_3,\; (1-\alpha)\rho_2 q_2^2 - \rho_3 q_3^2 = -(1-\alpha)p_2 + p_3 - F,\\
(1-\alpha)\rho_2 q_2^3 - \rho_3 q_3^3 + \frac{2\varkappa}{\varkappa-1}(1-\alpha)p_2 q_2 - \frac{2\varkappa}{\varkappa-1}p_3 q_3 = 0,\; p_1 = p_2,\; q_1 = q_2,
\end{gathered}
\tag{2}
$$

where U_0 is the velocity of the incident shock; V, velocity of the reflected shock; U_1, velocity of the passed shock; and F, loss of momentum.

The remaining notation is standard; the subscript corresponds to the zone number (see Fig. 2).

The system of equations (2) was solved by two methods. In the first the experimentally measured velocity of the shock penetrating the obstacle is substituted into the system (2). As a result of solving (2) we obtain the gas flow parameters in all the zones. To close the system (2) in the second, the velocity of the reflected shock V is taken from experiment in place of U_1.

The results of processing the experimental data, including that obtained in solving the system (2), are presented in Figs. 3 and 4. The circles correspond to experimental values of p_1/p_0 (a) and p_3/p_0 (b); the squares, the flagged figure, and the rhombi are used for results obtained by the theory of the "decay of the discontinuity," by a numerical computation (see below) and by a computation by the CCW theory; $\alpha = 0.09259$, 0.2941, 0.5556 respectively, for the unshaded, partially, and completely shaded points.

It must be kept in mind in examining the results presented that the error in experiment is superposed (the errors in determining the incident wave parameter, the time of the photograph, and the error in measuring the location of the front). Analysis of the error yields an estimate of 5%. The fact that the phenomenon corresponds only approximately to the idealized pattern (see Fig. 2) is also of definite value for the accuracy of the computation.

Comparing these results with the data of a computation can turn out to be interesting. Besides the application of specialized programs [2], utilization of general programs of the type in [5, 6] can turn out to be sufficiently productive. In the case when details of the flow in the neighborhood of x_0 are solved, its results within the framework of the ideal gas model evidently contain effects resulting in a limit representation, to the conception of the discontinuity of momentum. In the case when the homogeneous approximation deprives us of the possibility of a detailed computation of the flow around an element, the properties of the computation model become unclear and a comparison between the data of such a computation and a more definite result, experiment, say, is desirable.

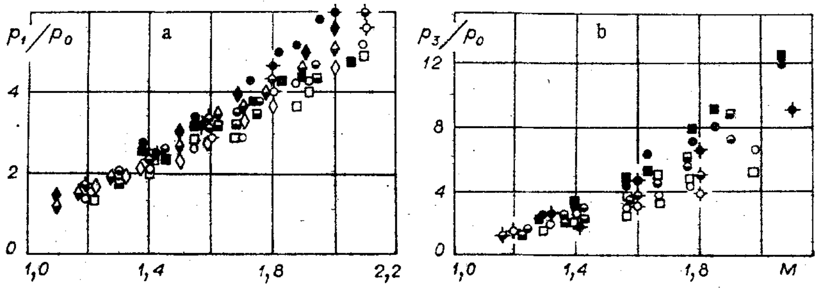

Fig. 4

The possibility of using the homogeneous approximation represented by the scheme is related, in principle, to the conception of partial cells, whose properties are formalized by the set of parameter $\{f_{ij}, A_{ijk}\}$ (the notation is the same as in [5]), where f_{ij} is the relative volume of a cell free for the flow, and A_k is the part of the k-th boundary of the cell open to the flow.

Let us consider the difference equations of the computation scheme by turning attention to the effect of this conception on the computation stages. To do this we assume that a cell arriving in the neighborhood of x_0 is placed in the field of view.

In the first stage, in conformity with an equation of the type

$$\tilde{U}_{ij}^n = U_{ij}^n - \frac{htA_{ij}^n}{\rho_{ij}^n f_{ij} hx}\left[(p+q)^n_{i+\frac{1}{2},\,j} - (p-q)^n_{i-\frac{1}{2},\,j}\right] \tag{3}$$

the change in velocity because of the existing inequality between the pressures ahead of the mouth and in the channel is taken into account.

The second computational step is to compute the mass flux through the parts of the cell boundaries open to the flow and the change it causes in the mean density

$$\rho_{ij}^{n+1} = \rho_{ij}^n + \frac{1}{V_j f_{ij}}\left(-\Delta M_{i+1/2,\,j} + \Delta M^n_{i,j-\frac{1}{2}} + \Delta M^n_{i-\frac{1}{2},\,j} - \Delta M^n_{i,j+\frac{1}{2}}\right), \tag{4}$$
$$\Delta M_{i+1/2,\,j} = A_{i+1/2,\,j} S_i^x \rho^n_{\ldots j} \tilde{U}^n_{i+1/2,\,j} h_t.$$

In the third stage the mean values of the velocity and internal energy components in the cell are determined from the flux balance

$$F_{ij}^{n+1} = \frac{1}{V_j f_{ij}\rho_{ij}^{n+1}}\left\{F_{ij}^n\rho_{ij}^n f_{ij} V_i + T_{ij}(1)\tilde{F}^n_{i-1,\,j}\Delta M^n_{i-\frac{1}{2},\,j} + [1 - T_{ij}(1)]\,\Delta M^n_{i-\frac{1}{2},j}\ldots\right\}. \tag{5}$$

Analyzing the computational scheme represented here by (3)-(5), it can be seen in what manner the usual property for a mesh approximation, which is the indistinguishability of details in the dimension O(h), is expressed in this specific case. The specific set (1) can evidently be set in correspondence with the set of modifications in the obstacle geometry, and the solution corresponds uniquely to this set.

This is the above-designated unclearness relative to the nature of the description of the scheme of gas-dynamic conditions at the leading edges of a homogeneous permeable obstacle.

Let us turn to the results of computations. Data obtained in a computation by the scheme discussed above are noted in Figs. 3 and 4. Data computed by the CCW theory [4] are superposed in these same graphs. As is seen, this theory which has been derived by using hypotheses about the smallness of α yields fair results even in other cases.

It should be noted that the computation and experiment, each on its own, become especially difficult and inaccurate as α approaches one.

In conclusion, it can be noted that the extensive experimental material obtained confirms the efficiency of the one-dimensional model of the phenomenon, and the closeness of its wave pattern to a central pattern, which indicates the unimportance (in the scale of the experiment) of processes with dissipation. It is interesting that both the computational method of "coarse particles" and the approximate CCW theory, which is somewhat formally used in this problem, are handled mainly with functions to describe shock interaction with a permeable obstacle in a homogeneous approximation.

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REGIMES AND PROPERTIES OF THREE-DIMENSIONAL SEPARATION FLOWS INITIATED BY SKEWED COMPRESSION SHOCKS

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The study of singularities in the three-dimensional interaction between an oblique compression shock and the boundary layer on a plane surface normal to the plane of the shock and parallel to the free-stream velocity vector is quite important to the comprehension of regularities in the origination and development of three-dimensional separation flows.

Sufficiently extensive information about the different properties of the flows under consideration is contained in published papers. Thus an analysis is made in [1] of known data on the three-dimensional interaction between an oblique shock and laminar and turbulent boundary layers. The results in this paper, together with [2], yield a representation of the flow structure at different stages of the interaction, from the start of separation into return flow. Correlation relationships are obtained to determine conditions for the origination of the separation flow [3] and the formation of secondary separation [4], as well as for the computation of the characteristic pressures and heat fluxes in the separation zones [5]. Measurements of the fields of different parameters are performed in the interaction domain [6].

Systematic investigations of the dynamics of separation flow development and its structure [7, 8] should especially be noted, which radically extend existing representations of its properties and refine the phenomenological schemes proposed earlier. The possibility and development of a secondary separation zone of bounded extent is detected in the last two papers. It is shown that there is no secondary separation under these conditions in the return stream in the turbulent boundary layer domain, and the increae in its extent in the laminar flow domain as the shock intensity grows is related to fastening the transition because of the intensifying spread of the flow, which contributes to diminution of the return flow streamline extent. The analysis performed in these papers also permitted showing that the disappearance detected in [9] and the repeated origination of the secondary separation for sufficiently high shock intensities is associated with the transformation of conically subsonic into supersonic flow in the separation zone. An important deduction is made here that the duplicate origination of secondary flow should be determined by the state of the boundary layer in the return flow and by the intensity of the internal compression shock which will occur.

In order to refine further the features of oblique compression shock interaction with a turbulent boundary layer being developed on a flat surface (Fig. 1), experimental investigations are performed for the Mach num-

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